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$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{\cos mx} - \sqrt{\cos nx}}{x^2}$$

پاسخ:

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{\cos mx} - \sqrt{\cos nx}}{x^2} = \lim_{x \rightarrow 0} \frac{\sqrt[3]{\cos mx} - 1 - (\sqrt{\cos nx} - 1)}{x^2} =$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt[3]{\cos mx} - 1}{x^2} - \frac{\sqrt{\cos nx} - 1}{x^2} = \lim_{x \rightarrow 0} \frac{\sqrt[3]{\cos mx}^3 - 1^3}{x^2} \times \frac{1}{3} - \frac{\sqrt{\cos nx}^2 - 1^2}{x^2} \times \frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{\cos mx - 1}{x^2} \times \frac{1}{3} - \frac{\cos nx - 1}{x^2} \times \frac{1}{2} = \lim_{x \rightarrow 0} \frac{-2 \sin^2\left(\frac{mx}{2}\right)}{x^2} \times \frac{1}{3} - \frac{-2 \sin^2\left(\frac{nx}{2}\right)}{x^2} \times \frac{1}{2}$$

$$= -2 \times \left(\frac{1}{3} \times \left(\frac{m}{2}\right)^2 - \frac{1}{2} \times \left(\frac{n}{2}\right)^2 \right) = \frac{3n^2 - 2m^2}{12}$$